Technical Notes

Determination of the Maximum Transmitted Shock at the Interface between Two Condensed Media

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WHEN a shock wave is normally incident upon the interface between two media, two shock waves are formed One is transmitted to the second medium while the other is reflected back into the original medium The ratio of the transmitted shock pressure (P_t) to the incident shock pressure (P_i) is often computed[1–3] from the "impedance-mismatch" approximation

$$\frac{P_t}{P_i} \cong \frac{2\rho_t U_t}{\rho_t U_t + \rho_i U_i} \tag{1}$$

which is strictly valid only for acoustic waves In Eq (1) $\rho_t U_t$ and $\rho_i U_i$ are the shock impedances (density times shock velocity) of the transmitted and incident media, respectively From Eq (1), for $\rho_t U_t \gg \rho_i U_i$, it has been assumed that the maximum value of P_t/P_i is two However, examination of the true "impedance-mismatch" relation

$$\frac{P_t}{P_i} = \frac{1 + (\rho_r U_r/\rho_i U_i)}{1 + (\rho_r U_r/\rho_t U_t)} \tag{2}$$

shows that if $\rho_t U_t \gg \rho_r U_r \gg \rho_i U_i$, (P_t/P_i) may be greater than two Unfortunately, the maximum value of (P_t/P_i) cannot be deduced from Eq (2)

Using the graphical Hugoniot "reflection" method[4–8] an estimate of $(P_t/P_i)_{\max}$ may be made In this method, the incident medium Hugoniot is plotted on the pressure-particle velocity (P-u) plane and reflected 180° around a vertical line drawn through the point of intersection of the incident pressure (P_i) and the incident medium Hugoniot The transmitted pressure (P_t) is then determined by the intersection of this "reflected" Hugoniot with the Hugoniot of the acceptor material Ideally, the maximum value of P_t for the given value of P_i will occur when the acceptor material has as its Hugoniot a vertical line at the origin (i e , $\rho_t \rightarrow \infty$) This is visualized in Fig 1 From symmetry, the "reflected" Hugoniot is seen to intersect the abscissa at $2u_i$

In many cases, the incident medium Hugoniot may be written in quadratic form[2 6 8]:

$$P = au^2 + bu \tag{3}$$

where a and b are constants Since the "reflected" Hugoniot is simply a reflection of Eq (3), it may also be written in quadratic form However, since the "reflected" Hugoniot does not pass through the origin it must be written more generally as

$$P_{\text{reflected}} = cu^2 + du + e \tag{4}$$

where c, d, and e are constants The boundary conditions pertaining to Eqs (3) and (4) can be used to relate c, d, and e to a and b These conditions may be deduced from Fig 1 and are: 1) when $u = 0$, $P_{\text{reflected}} = P$ at ($u = 2u_i$); 2) when $u = u_i$, $P_{\text{reflected}} = P$, and 3) when $u = 2u_i$, $P_{\text{reflected}} = 0$ Applying these conditions gives

$$e = a(2u_i)^2 + b(2u_i) \tag{5}$$

$$c(u_i^2) + d(u_i) + e = a(u_i^2) + b(u_i) \tag{6}$$

$$c(2u_i)^2 + d(2u_i) + e = 0 \tag{7}$$

which are simultaneous equations in c, d, and e The solutions are

$$c = a \tag{8}$$

$$d = -4au_i - b \tag{9}$$

$$e = 4au_i^2 + 2\,bu_i \tag{10}$$

Substituting these into Eq (4) and rearranging gives

$$P_{\text{reflected}} = a(2u_i - u)^2 + b(2u_i - u) \tag{11}$$

From the diagram it is clear that $P_{t\max}$ is given by $P_{\text{reflected}}$ when $u = 0$ Thus

$$P_{t\max} = 4au_i^2 + 2bu_i \tag{12}$$

Also, P_i is found from Eq (3) when $u = u_i$, and thus

$$P_i = au_i^2 + bu_i \tag{13}$$

Forming the ratio of Eqs (12) and (13) gives

$$\frac{P_{t\max}}{P_i} = \frac{4au_i^2 + 2bu_i}{au_i^2 + bu_i} = \frac{4au_i + 2b}{au_i + b} \tag{14}$$

This represents the maximum value of P_t/P_i for a given value of u_i However, it is desired to compute the maximum of P_t/P_i for *any* value of u_i Since u_i can have any value on the abscissa, it may be replaced by u Eq (14) becomes

$$P_t/P_i = (4au + 2b)/(au + b) \tag{15}$$

The maximum value of this function is found by determining the derivative with respect to u and setting the resulting expression equal to zero; therefore,

$$\frac{d(P_t/P_i)}{du} = \frac{2ab}{(au + b)^2} = 0 \tag{16}$$

from which it may be deduced that as $u \rightarrow \infty$, $(P_t/P_i) \rightarrow$ maximum (the second derivative is negative) Therefore,

$$\left(\frac{P_t}{P_i}\right)_{\max} = \lim_{u\rightarrow\infty}\left(\frac{4au + b}{au + b}\right) = \lim_{u\rightarrow\infty}\left(\frac{4a + (b/u)}{a + (b/u)}\right) = \frac{4a}{a} \tag{17}$$

or

$$(P_t/P_i)_{\max} = 4 \tag{18}$$

It should be noted that this result depends directly on the fact that the Hugoniot of the incident medium (in this case Plexiglas) can be expressed as a quadratic in u This in turn depends on the assumption (empirically supported)[2 6 8] that the U-u curve of the medium is linear Because it is not known if this relation stays linear as u becomes very large, the result in Eq (18) may be considered semiempirical in nature However, if the "true" Hugoniot can be expressed in a polynomial of any degree, $(P_t/P_i)_{\max}$ can always be found in a manner similar to that shown herein For materials with a Hugoniot expressible as a cubic equation,[6] the fore-

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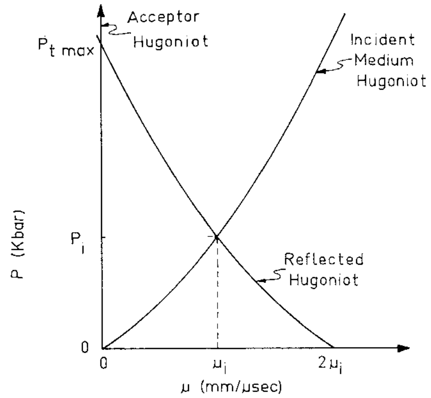

Fig 1 Hugoniot "reflection" method

mentioned approach gives $(P_t/P_i)_{max} = 8$ It should also be pointed out that the result also depends on the inherent accuracy of the Hugoniot "reflection" method itself

In order to illustrate some of the foregoing remarks, (P_t/P_i) was computed for shock transmission from Plexiglas[9] to iron[10] and Plexiglas to platinum[10] using both the "impedance-mismatch" approximation [Eq (1)] and the Hugoniot "reflection" method The results are shown in columns 1–5 of Table 1 It is seen that Eq (1) gives quite low values in this region and that the ratio can, as indicated, be greater than two at finite pressures in real materials Columns 6 and 7 in Table 1 show the values of $(P_{t\ max}/P_i)$ as computed from Eq (14) compared to the maximum value from Eq (1) Clearly, increases in P_i give increases in $(P_{t\ max}/P_i)$, approaching the limiting value of 4 computed in Eq (18)

Table 1 Comparison of values of (P_t/P_i) for shock transmission from Plexiglas to iron and platinum computed by the "impedance-mismatch" approximation and the Hugoniot "reflection" method

Plexiglas $\rho_i = 1\ 18$ g/cm³	Iron $\rho_t = 7\ 84$ g/cm³		Platinum $\rho_t = 21\ 37$ g/cm³		Ordinate $\rho_t \to \infty$	
P_i, kbar	$\left(\frac{P_t}{P_i}\right)^a$	$\left(\frac{P_t}{P_i}\right)^b$	$\left(\frac{P_t}{P_i}\right)^a$	$\left(\frac{P_t}{P_i}\right)^b$	$\left(\frac{P_t}{P_i}\right)^a$	$\left(\frac{P_{t\max}}{P_i}\right)^c$
102 7	1 70	2 14	1 86	2 61	2 00	3 13
162 4	1 66	2 12	1 83	2 56	2 00	3 25
235 0	1 65	2 11	1 83	2 60	2 00	3 35

[a] Computed from impedance mismatch approximation Eq (1)
[b] Computed by Hugoniot "reflection method
[c] Computed from Eq (14)

References

[1] Cook, M A and Udy, L L, "Calibrations of the card-gap test," ARS J **31**, 52–57 (1961)

[2] Jaffe, I, Beauregard, R, and Amster, A, "Determination of the shock pressure required to initiate detonation of an acceptor in the shock sensitivity test," ARS J **32**, 22–25 (1962)

[3] Deal, W E, "Measurement of Chapman-Jouget pressure for explosives," J Chem Phys **27**, 796–800 (1957)

[4] Price, D and Jaffe, I, "Large scale gap-tests: interpretation of results for propellants, ' ARS J **31**, 595 (1961)

[5] Walsh, J M and Rice, M H, "Dynamic compression of liquids from measurements on strong shock waves," J Chem Phys **26**, 815 (1957)

[6] Rice, M H, McQueen, R G, and Walsh, J M, "Compression of solids by strong shock waves," *Solid State Physics*, edited by F Seitz and D Turnball (Academic Press Inc, New York, 1958), Vol 6, pp 1–63

[7] Rice, M H and Walsh, J M, "Equation of state of water to 250 kilobars," J Chem Phys **26**, 824 (1957)

[8] Al'tshuler, L V, Kropnikov, K K, and Brazhnik, M I, "Dynamic compressibilities of metals under pressures from 400,000 to 4,000,000 atmospheres" Soviet Phys —JETP **34**(7), 614 (1958)

[9] Salzman, P K, "Analysis of shock attenuation for 0 5 and 2 0 in diameter card-gap sensitivity tests, ' ARS Preprint 2344-A-62 (January 1962)

[10] Walsh, J M, Rice, M H, McQueen R G, and Yarger, F L, ' Shock-wave compressions of twenty-seven metals Equations of state of metals,' Phys Rev **108**, 196–216 (1957)

Stability of the Hodograph Equations in One-Dimensional Reacting Gas Flow

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IT can be shown that the standard equations describing flow of reacting gases in nozzles have singularities at the Mach number equal to unity Various methods have been devised for passing through this point, including extrapolation[1] and lowering of computation accuracy to allow the calculations to skip from the subsonic to supersonic region [2]

When the equations are written in hodograph form, using the velocity as independent variable, this singularity no longer occurs at $M_f = 1$, and one could hopefully integrate with complete confidence in this region Furthermore, in the subsonic portion of the nozzle, the rates of change of the gross properties with distance are quite large, and frequently a small mesh size is required The design computation time might be reduced significantly by using the hodograph form of the equations

However, the author has found that the nonequilibrium equations are unstable in many cases This note, then, is an attempt to establish a stability criterion for these equations As an illustration, the simple case of dissociating gas flow will be considered

Consider the system of equations describing the flow of an $A_2 \gtrless 2A_1$ gas in a nozzle Assume that the velocity can be used as the independent variable The equations are as follows:

Continuity

$$\frac{1}{\rho}\frac{d\rho}{du} + \frac{1}{A}\frac{dA}{du} + \frac{1}{u} = 0 \tag{1}$$

Momentum

$$\frac{dP}{du} = -\rho u \tag{2}$$

Energy

$$\frac{\Delta H}{M_2}\frac{d\alpha}{du} + \frac{c_P}{M_2}\frac{dT}{du} + u = 0 \tag{3}$$

State

$$\frac{1}{P}\frac{dP}{du} = \frac{1}{\rho}\frac{d\rho}{du} + \frac{1}{T}\frac{dT}{du} + \frac{1}{1+\alpha}\frac{d\alpha}{du} \tag{4}$$

Nozzle Shape

$$\frac{d\ \ln A}{du} = F(x)\frac{dx}{du} \tag{5}$$

Kinetic Rate

$$u\frac{d\alpha}{du} = k\left\{\frac{\alpha_e^2 - \alpha^2}{\alpha^2(1+\alpha)}\right\}\frac{dx}{du} = k\Delta\frac{dx}{du} \tag{6}$$

where α is the extent of dissociation of the dimer, α_e is the

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